DYNAMIC BALANCING OF RIGID ROTORS BY THE INFLUENCE COEFFICIENT METHOD

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ABTRACT

Applied to rigid rotors, the influence coefficient method is the most adaptable method for rotor dynamic balancing because it uses only experimental information. The sole requirement for balancing rotors with this method is that there exists proportionality between mass unbalance located at correction planes and vibration measured at measuring planes. Hence, the rotors only need to be described by correction and measuring planes. The sensitivity of the measuring planes compared to the correction planes determines the response of the amount of unbalance for a specified rotation speed. The paper aims to describe in detail the experiment for dynamic balancing of rotors to verify the influence coefficient method. The direct calibration on two-plane rigid rotor using this method showed that a significant reduction of unbalances can be obtained. The results can be helpful for the design and safety assessment of dynamic systems equipped with rotors for marine engineering.

Keywords: *Rotor dynamic balancing, influence coefficient method, residual unbalances, Discrete Fourier Transform*

I. INTRODUCTION

The mass unbalance is usually the primary cause of vibration in rotating equipment where dynamic forces excite structures, machines, vehicles including the engine room of fishing boats... The effects of vibration may be wear, reduced performance, faulty operation, or any degree of irreversible damage. The purpose of rotor balancing is to redistribute the mass of a rotor so that it creates less centrifugal forces due to unbalance on its bearings. This aim can be attained only to a certain degree as, even after balancing, residual unbalance will inevitably remain. In balancing rigid rotors, the *influence coefficient* (IC) method is a reliable balancing method that is used extensively today [1, 2]. It can be applied effectively for balancing rotors both on balancing benches and on site [3]. The principle of this method is to apply trial masses at rotor balancing planes to obtain all influence coefficients (ICs) between the rotor balancing planes and the bearings where installed the vibration transducers. The IC method leaves out of consideration the type of balancing machine and transducers, shape of the rotor, or its deformation. When the balancing program is used at the calibration stage, the instrumentation should be calibrated in three test runs for 2-plane rotors. In direct calibration for 1-plane rotors, the calibration procedure is identical to calibration on two planes but the operation is carried out with just two calibration runs. The calibration values can be memorized and reused when balancing under similar conditions. For the balancing method to function correctly in each case, it is essential for the calibration runs and balancing of subsequent rotors to be performed at the same speed of rotation. Hence the balancing speed used in calibration is memorized so that during use, even after some time, the speed at which to perform the test is known from the memorized data. The counterweight used in testing should be chosen depending on the entity of the presumed unbalance of the rotor. The study results can be applied to the efficient design and operation of various rotating equipment, including marine engine rooms and fisheries production.

II. RIGID ROTORS BALANCING MODEL

Balancing a rotor can be considered in two different ways, on a balancing ma-chine, and on site [3]. When the rotor can be removed from the machine of which it is a part, its balancing can be carried out on a bench specially designed for this purpose, also known as a dynamic balancing machine. In this way, the rotor is balanced on its bearings (of elasticity in general unknown) at the nominal rota-tional speed. On-site balancing is also a practical solution for rotor balancing but a minimum number of balancing trial runs must be done. Although there are two types of balancing machines: hard bearing and soft bearing balancing machines. The first one can indicate directly, after a measuring run, the unbalance and its position. A quick presetting of certain rotor dimensions is necessary to obtain such results. In the second case, balancing is carried out at a rotational speed that sufficiently exceeds the range of critical speeds.

2.1 Measuring and balancing planes

The dynamic balancing machine measures unbalances in two planes in which the rotor is supported (usually with bearings). However, the unbalance corrections are made in some other places where it is convenient to add or remove weights [4]. The former is called *measuring plane*, while the latter is known as *correction plane* or *balancing plane*. Consider the system in **Figure 1**. It consists of a rigid rotor placed on two elastic bearings A and B. There are two balancing planes *i* and *ii* in which one can make the mass corrections necessary for the cancellation of the vibrations at the bearings.

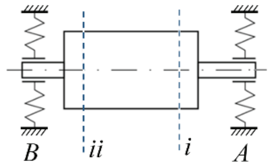


Figure 1. Dynamic balancing on two-plane rotor simultaneously.

2.2 Proportionality between unbalance and vibration

The balancing process requires finding the unbalances at the correction planes and then attaching the necessary counter weights to reduce the vibration at bearings. This can be done based on the results of the analysis of the vibration measurement thanks to 2 vibration transducers and 1 phase sensor (phasor). The most commonly used device for vibration measurement in the balancing machine is the piezo-electric force transducer, which gives an electrical signal proportional to the vibration. This signal can readily be amplified and analyzed. These vibration quantities are usually expressed as voltages $\boldsymbol{V}_{1,2}$ and measured thanks to the vibration transducers integrated in bearings *A* and *B*. $\boldsymbol{V}_{1,2}$ are found proportional to the unbalances if we admit the linearity of this electromechanical system.

One represents in complex notation (amplitude and phase) the following quantities:

$\boldsymbol{D}_{1,2}$ - initial unbalances on correction planes *i*, *ii*;

$\boldsymbol{V}_{1,2}$ - vibration measured at the bearing *A*, *B*.

The initial unbalances $\boldsymbol{D}_{1,2}$ produce the corresponding vibrations in 2 planes *A*, *B* and generate the proportional voltages $\boldsymbol{V}_{1,2}$ measured by transducers (**Figure 2**):

$$\boldsymbol{D}_{1,2} = D_{1,2}\mathrm{e}^{i(\omega t+\theta_{1,2})} = D_{1,2}(\cos(\omega t + \theta_{1,2}) + i\sin(\omega t + \theta_{1,2})) \quad (1)$$

$$\boldsymbol{V}_{1,2} = V_{1,2}\mathrm{e}^{i(\omega t+\phi_{1,2})} = V_{1,2}(\cos(\omega t + \phi_{1,2}) + i\sin(\omega t + \phi_{1,2})) \quad (2)$$

with the unbalance magnitudes:

$$D_{1,2} = m_{1,2}r_{1,2} \quad (3)$$

where $m_{1,2}$, $r_{1,2}$ - unbalance masses and corresponding radii; $\theta_{1,2}$ - unbalance phases; $\varphi_{1,2}$ - proportional voltage phases, ω - rotational speed of the rotor (balancing speed).

Note that $\boldsymbol{V}_{1,2}$ measured at the bearing are projected horizontally since the nature of the measurement of vibrations at bearings. It corresponds to the real part of $\boldsymbol{V}_{1,2}$:

$$\mathcal{Re}(\boldsymbol{V}_{1,2}\mathrm{e}^{i(\omega t+\phi_{1,2})}) = V_{1,2}(\cos(\omega t + \phi_{1,2})) \quad (4)$$

Similarly, the trial unbalance using a trial mass placed on the plane *i* or *ii* at a pre-selected radius is also expressed as follows:

$$\boldsymbol{M}_{1,2} = M_{1,2}e^{i(\psi_{1,2})} = M_{1,2}(\cos\psi_{1,2} + i\sin\psi_{1,2}) \quad (5)$$

with the trial unbalance magnitudes:

$$M_{1,2} = \overline{m}_{1,2}\overline{r}_{1,2} \quad (6)$$

where $\overline{m}_{1,2}\overline{r}_{1,2}$ trial masses and corresponding radii; $\psi_{1,2}$ - trial unbalance phases.

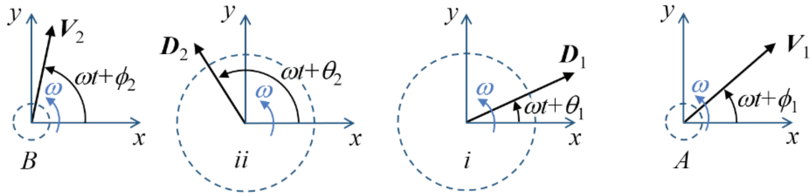


Figure 2. Initial unbalances and proportional voltages.

III. VIBRATION MEASUREMENTS

The hard bearing balancing machine is essentially an elastic mechanical system that attaches a rotating object that needs to be balanced. This system has natural frequencies that are very high compared to the balancing rotation frequencies. When rotating, the rotor creates centrifugal forces that act as excitation forces to the elastic system. Thanks to the integrated measurement system with vibration transducers, phasor, the vibration characteristics of the system (rotation frequency, amplitudes, phases) are determined.

3.1 Dynamic balancing test bench

The typical layout of a hard bearing balancing machine is shown in **Figure 3**. Rotor (3) is mounted on 2 vibration supports having 2 vibration transducers integrated inside (4). Electric motor *M* (1) spins rotor thanks to belt drive (2). The main phase of the rotor is determined by the phase disc on which the reference mark (5) is applied and the phase sensor (photoelectric cell phasor) (6). It is also used to measure rotation speed. The vibration signals measured from the supports and phase signals are processed in real time by the analyzer (usually with a computer) (9) through the data acquisition card as well as the control card (7). The rotor speed is controlled through the inverter (8) according to the rotation speed also measured from (6).

3.2 Discrete Fourier Transform as noise filter

In signal, noises such as DC component, high-order harmonic, random noise, etc. are always present and it makes the vibration measurement might get inaccurate (**Figure 4**, top). It is necessary to remove the noises from the source by extracting only fundamental component of the vibration signal corresponding to rotor spinning frequency. The Discrete Fourier Transform (DFT) method can be used in rejecting higher frequency harmonic signals effectively, i.e. filter them out. It can be used as a numerical filter and have also been applied for the signal to get optimum results. DFT is the transform that deals with a finite discrete-time signal and a finite or discrete number of frequencies [5, 6]. Assume that the time signal $v(t)$ is recorded over a finite period of time T with an evenly sampling time Δt and N samples per cycle of the signal, hence $\Delta t = T/N$. Let $v[n]$, $n = 0, 1, 2,\ldots, N$-1, denotes an N-sample real-valued and finite length sequence. Then $v[n] = x(n\Delta t)$ is the value at the current sample n. Obviously, $\omega\Delta t = 2\pi/N$. The N-point DFT of any signal $v[n]$ is defined as follows [6]:

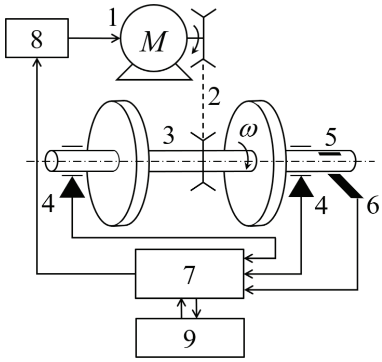


Figure 3. Dynamic balancing machine layout.

$$V[k] = \sum_{n=0}^{N-1} v[n]e^{-i\frac{2\pi}{N}nk}, \quad k = 0, 1, 2, \ldots, N-1 \quad (7)$$

The output $V[k]$ is a complex function that encodes both the amplitude and phase information of a complex sinusoidal component $e^{i \cdot 2\pi nk/N}$ of $x[n]$ at k^{th} frequency, ω_k:

$$\omega_k = \frac{2\pi}{T}k, \quad k = 0, 1, 2, \ldots N-1 \qquad (8)$$

If the input signal is a real-valued sequence, the amplitude and phase of the signal can be calculated as:

$$V[k] = \frac{|V_k|}{N/2} = \frac{\sqrt{Re(V_k)^2 + Im(V_k)^2}}{N/2} \qquad (9)$$

$$\phi[k] = \tan^{-1}(Im(V_k) / Re(V_k)) \qquad (10)$$

where $Re(V_k)$, $Im(V_k)$ are the real and imaginary parts of the complex number.

The proportional voltages $V_{1,2}$ usually accompany noise that needs to be removed by an appropriated noise filter. The N-point DFT of N samples $x[n]$ measured in real-time allows for the calculation of amplitude and phase of the vibration signals at the balancing rotation frequency. It also plays the role of the noise filter when only $\{V_{1,2}\}_{k=1}$ is taken into account, i.e. only the fundamental frequency signals ($\omega_1 = 2\pi/T$) under the same condition are recorded. In other words, one can recover the original sequence $x[n]$ but without noise, because only the fundamental component of the vibration signal corresponding to the rotor spinning frequency is extracted. Besides, the measurement of phase is essential in the rotor balancing [7]. It always exists the dephase (phase difference) between the fundamental frequency vibration signal and the determined reference mark on the shaft. These dephase values can be determined experimentally by using a proving rotor (considered as a free unbalance rotor) with a reference mark on it. The phase of each harmonic is also available by DFT formula (Eq.10) and hence also for the fundamental frequency, i.e. balancing rotation frequency.

Note that moving the phase of $V_{1,2}$ by 90°

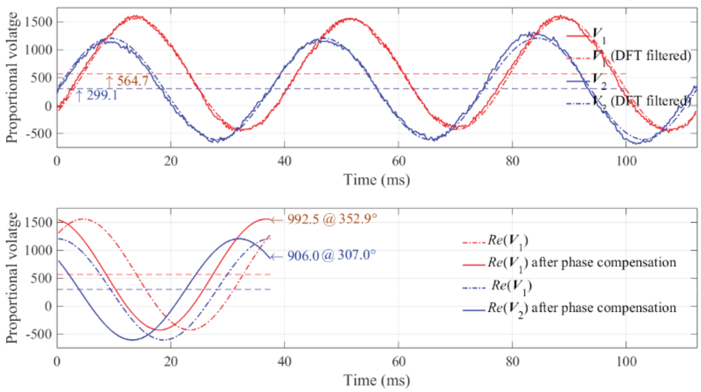


Figure 4. Vibration signals before and after noise filterting and phase compensation.

means taking only its real part $Re(V_{1,2})$, i.e. only horizontal component of the signal is considered. Compensate for the dephase, one has the final amplitude and phase on each measuring plane (**Figure 4**, bottom). One could Figure out that when DFT filter is applied, the effect of DC component, high-order harmonic, and random noise have been ignored.

IV. INFLUENCE COEFFICIENT METHOD

The main idea of the IC method is to use a reference run with the rotor in the initial configuration and two more trial runs with a trial mass attached to the rotor for each balancing plane to generate the ICs [2, 4, 8, 9]. The relationship between the initial unbalances, trial masses at the balancing planes, and the corresponding vibration forces in the measuring planes are derived by the proposed IC method. When balancing a rotor on its bearings, one must simultaneously measure the vibrations in two bearings under the effect of the initial unbalances in both balancing planes, and then with trial mass. An IC model for a rigid rotor relates measured vibration V_{kl} to unbalances D_i and trial unbalance M_i in the successive tests. By making the hypothesis of linearity of the system, one can write a proportionality relationship between the vibrations measured at the bearings and the unbalances without or with trial unbalance at each balancing plane:

$$\boldsymbol{V}_{kl} = \sum_{i=1}^{2} \boldsymbol{\alpha}_{ki}(\boldsymbol{D}_i + \boldsymbol{M}_i) \quad k = 1,2,\ l = 0,1,2. \quad (11)$$

where:

$\boldsymbol{V}_{\text{kl}}$ denotes the vibration measured at the k bearing in the test l, the value $l = 0$ corresponding to a test of the rotor in initial configuration (with initial unbalances), i.e. at first run and without any trial mass on rotor; the value $l = 1, 2$ corresponding to initial unbalances with trial unbalance mounted on plane i and ii sequentially.

$\boldsymbol{\alpha}_{\text{ki}}$ denotes the complex ICs of proportionality between the unbalances and the vibration it causes. It should be noted that $\boldsymbol{\alpha}_{\text{ki}}$ are functions of rotational speed and depend on the system characteristics.

Three successive tests are carried out:

Step 1. Rotor in initial configuration, with the $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ to be determined in planes i and ii. The vibrations $\boldsymbol{V}_{10}$ and $\boldsymbol{V}_{20}$ are recorded at the bearings A and B.

Step 2. A trial unbalance $\boldsymbol{M}_1$ is then installed in plane i, and the vibrations $\boldsymbol{V}_{11}$ and $\boldsymbol{V}_{21}$ are recorded at the bearings A and B.

Step 3. A trial unbalance $\boldsymbol{M}_2$ is finally installed in plane ii after removing $\boldsymbol{M}_1$, and the vibrations $\boldsymbol{V}_{12}$ and $\boldsymbol{V}_{22}$ are recorded at the bearings A and B.

The relations in Eq (11) allow to calculate all ICs and the unbalances $\boldsymbol{D}_{1,2}$. If the vibrations $\boldsymbol{V}_{\text{kl}}$ - $\boldsymbol{V}_{\text{k0}}$ due to the trial masses alone are calculated, one obtains:

$$\boldsymbol{\alpha}_{ki} = \frac{\boldsymbol{V}_{ki} - \boldsymbol{V}_{k0}}{\boldsymbol{M}_i} \quad (12)$$

with their x- and y- components corresponding to the real and imaginary parts of $\boldsymbol{\alpha}_{\text{ki}}$:

$$\alpha_{ki,x} = \mathcal{R}e(\boldsymbol{V}_{ki} - \boldsymbol{V}_{k0} / \boldsymbol{M}_i) \quad (13)$$

$$\alpha_{ki,y} = Im(\boldsymbol{V}_{ki} - \boldsymbol{V}_{k0} / \boldsymbol{M}_i) \quad (14)$$

Solving for $\boldsymbol{D}_{1,2}$ gives the initial unbalances:

$$\begin{pmatrix} \boldsymbol{D}_1 \\ \boldsymbol{D}_2 \end{pmatrix} = \begin{bmatrix} \boldsymbol{\alpha}_{11} & \boldsymbol{\alpha}_{12} \\ \boldsymbol{\alpha}_{21} & \boldsymbol{\alpha}_{22} \end{bmatrix}^{-1} \begin{pmatrix} \boldsymbol{V}_{10} \\ \boldsymbol{V}_{20} \end{pmatrix} \quad (15)$$

The expressions obtained for the unbalances shows that the experimental determination requires 3 successive measurements of the vibration in planes A and B, before and after the addition of 2 trial masses. The magnitudes of the unbalances and the phases or position angles of these unbalances $D_{1,2}$ and $\theta_{1,2}$ respectively, can be calculated as:

$$D_{1,2} = \left|\boldsymbol{D}_{1,2}\right| = \sqrt{D_{1,2x}^2 + D_{1,2y}^2} \quad (16)$$

$$\theta_{1,2} = \tan^{-1}(D_{1,2y} / D_{1,2x}) \quad (17)$$

that we can put back the unbalances in the polar form: $D_{1,2}e^{i\theta 1,2}$ with $D_{1,2}$ and $\theta_{1,2}$ given by Eqs. (16), (17) respectively. The correction masses to be added to the correction planes depend on the radius of $r_{1,2}$ preselected, so $m_{1,2} = D_{1,2}/r_{1,2}$ at angles $\theta_{1,2} + 180°$ as counterweights. The balancing procedure for a 2-plane rotor based on this method is illustrated in **Figure 5** with 2 distinct stages: calibration (a) and direct balancing (b).

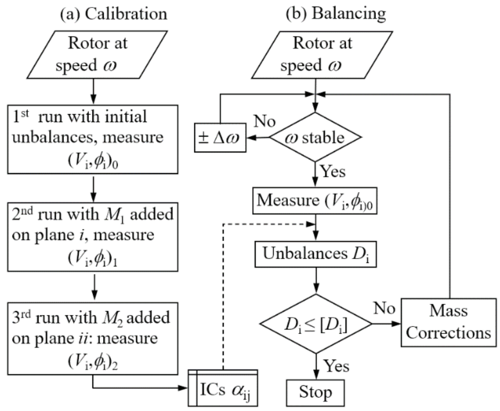


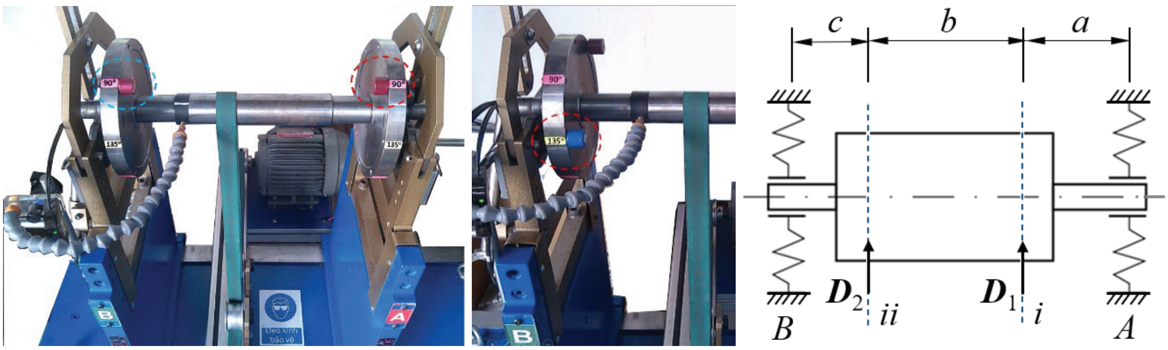
Figure 5. Two stages of calibration (a) and balancing (b)

V. EXPERIMENTAL VERIFICATION

The actual experimental rotor balancing is carried out on a proving rotor of weight 15 kg considered as a 2-plane rigid rotor. This rotor can also be used to calibrate a hard-bearing balancing machine as well as to assess its sensitivity. The dephase values between the fundamental vibration signals and the determined reference mark on the shaft are also determined experimentally, before the calibration stage of balancing of a rotor.

5.1 Experimental setup

The experiments to generate ICs are performed on the dynamic balancing test bench HnB75B. It was developed to perform balancing of rotor easily and accurately as a real belt-driven dynamic balancing machine. A long, rigid rotor is supported in its bearings and is driven by a motor (**Figure 6**) whose speed can be controlled as shown in **Figure 3**. Vibration amplitudes and phases are available by DFT formula applying for the vibration measurement at each test run without or with trial mass. Then according to the ICs, the unbalance corrections to eliminate the initial vibration can be computed. All tests are performed at the same speed of rotation.



(a) (b) (c)

Figure 6. Proving rotor with 2 initial unbalance masses (a) and with 1 trial mass(b) mounted on HnB75B between 2 supports (c).

Setup parameters:

- Rotor dimensions (**Figure 6**, c): a = 50 mm, b = 300 mm, c = 50 mm;
- Radius to apply trial masses, correction masses: r = 85 mm;
- Measurement parameters: Number of samples: N = 512; Sampling rate: f_{samp} = 3 kHz;
- Balancing speed: RPM 1200;
- Trial mass: 20 g;
- Maximum permissible residual unbalance per unit of rotor weight [g·mm/kg] or balancing tolerance according to the standard ISO 1940/1 [10] at balancing quality grade of G2.5 under operation speed RPM 1200:

Tolerance [*D*] = *Rotor weight* [kg] × *Quality grade* (G) × 9549 / *Operational Speed* [RPM]
= 15 × 2.5 × 9549 / 1200 = 19.89 g·mm/kg (for both 2 planes).

Each plane of rotor of 15 kg at radius r = 85 mm has then the maximum permissible unbalance mass: [m] = ([D] × 15/85) / 2 = 1.76 g.

The procedure for the experimental data collection is as follows:

1. Attach 2 masses as initial unbalances on 2 planes i and ii of the proving rotor (**Figure 6**, a): $\bar{m}_1 = \bar{m}_2 = 20$g, $\bar{\theta}_1 = \bar{\theta}_2 = 0°$ at the same radius r = 85 mm.

2. For each test run, carry out two sets of vibration measurements at bearing positions corresponding to (1) initial unbalances $(\bar{m}_i, \bar{\theta}_i)$; (2) initial unbalances with trial unbalance at plane i (M_1, ψ_1); (3) initial unbalances with trial unbalance (**Figure 6**, b) at plane ii (M_2,

ψ_2), then calculate their amplitudes and phases (Eqs. (9), (10)).

3. Compute ICs $\boldsymbol{\alpha}_{ij}$ (x- and y- components) and the unbalances (D_1, θ_1), (D_2, θ_2), then correction masses (m_1, θ_1 +180°), (m_2, θ_2 +180°).

Determination of ICs and unbalances

The experimental vibration data of 3 tests #1-3, and 3 successive test run results for each under the same rotation speed RPM 1200 are presented in **Table 1**.

The unbalance reduction rates of the rigid rotor reach respectively 99.2% and 97.3% for the test #1, 99.3% and 98.1% for the test #2, 95.1% and 95.9% for the test #3. In fact, the maximum residual unbalance mass is 0.98 g, achieving an unbalance reduction rate of over 95%. The unbalance correction mass deviation, i.e. the residual unbalance, is less than the maximum permissible unbalance mass ([m] = 1.76 g) satisfying the balancing quality grade of G2.5 (ISO 1940/1 [10]) with phase angle error < ±1°.

Table 1. Vibration measurements and unbalance corrections

TEST	Test Run	Initial Unbalances at r = 85 mm ($\bar{m}_1, \bar{\theta}_1$)	($\bar{m}_2, \bar{\theta}_2$)	Trial unbalance (M_i, ψ_i)	Vibration amplitudes & phases on planes A and B (V_1, ϕ_1)	(V_2, ϕ_2)	ICs $\boldsymbol{\alpha}_{ij}$, Unbalance corrections, ($m_1, \theta_1 + 180°$), ($m_2, \theta_2 + 180°$)	Residual unbalances (m_{Res}, θ_{Res}), (m_{Res}, θ_{Res})
#1	1	20g∠ 0°	20g∠ 0°	-	2568.2∠0.3°	3027.9∠0.2°	$\begin{bmatrix} 106.67 & 22.66 \\ 24.22 & 130.81 \end{bmatrix} + i$	$\begin{bmatrix} 106.77 & 20.21 \\ 24.06 & 126.68 \end{bmatrix}$
	2	20g∠0°	20g∠0°	20g∠135°	1857.4∠55.2°	2708.2∠7.4°	m_1, m_2	m_{Res}
	3	20g∠0°	20g∠0°	20g∠225°	2263.9∠353.1°	2135.3∠303.5°	19.94g∠180.4°, 19.46g∠180.2°	0.15g∠-66.5°, 0.54g∠-7.2°
#2	1	20g∠ 0°	20g∠ 0°	-	2568.1∠0.0°	3028.2∠0.4°	$\begin{bmatrix} 106.48 & 23.03 \\ 24.24 & 129.52 \end{bmatrix} + i$	$\begin{bmatrix} 107.18 & 19.84 \\ 23.20 & 128.94 \end{bmatrix}$
	2	20g∠0°	20g∠0°	20g∠135°	1849.8∠55.0°	2707.7∠7.4°	m_1, m_2	m_{Res}
	3	20g∠0°	20g∠0°	20g∠225°	2260.1∠352.8°	2165.5∠303.5°	19.87g∠179.9°, 19.66g∠180.4°	0.13g∠14.9°, 0.37g∠21.9°
#3	1	20g∠ 0°	20g∠ 0°	-	2585.9∠0.4°	3081.8∠0.7°	$\begin{bmatrix} 106.91 & 17.83 \\ 26.69 & 131.10 \end{bmatrix} + i$	$\begin{bmatrix} 107.63 & 19.13 \\ 25.32 & 131.76 \end{bmatrix}$
	2	20g∠0°	20g∠0°	20g∠135°	1876.1∠55.1°	2732.7∠8.3°	m_1, m_2	m_{Res}
	3	20g∠0°	20g∠0°	20g∠225°	2347.5∠353.8°	2200.8∠303.9°	20.98g∠180.3°, 19.24g∠180.8°	0.98g∠173.6°, 0.81g∠-19.4°

5.3 Measurement uncertainty

In this study, only the measurement uncertainty is concerned for vibration measurements in terms of proportional voltages, V_1 and V_2. The measuring system and the data acquisition software integrated in the balancing analyzer (**Figure 3**) enables displaying and recording the vibration signals measured at the supports. Although several measurements are taken during balancing calibration, only twelve measurements are presented for the first test (with initial unbalances) and under the same balancing speed. However, more measurements are preferred for the less uncertainty. In practice, there are many possible sources of uncertainty in a vibration measurement. However, the variations in repeated observations of the measurand under apparently identical conditions, i.e. under the same setup parameters mentioned above, should be firstly considered.

For Type-A uncertainty analysis [11], an estimate of the standard deviation of the distribution values $s(V_i)$ is given by:

$$s(V_i) = \sqrt{\frac{1}{n-1}\left[\sum_{j=1}^{n}(V_{i,j} - \bar{V}_i)^2\right]} \qquad (21)$$

where $\bar{V}_i$ is the mean value of n individual measured values $V_{i,j}$.

Table 2 shows the steps to calculate the Type-A uncertainty of the vibration measurements V_1, V_2. For n readings, the standard uncertainty (U_A) is estimated as following:

$$U_A(V_i) = \sqrt{\frac{1}{n}} s(V_i) \qquad (22)$$

Table 2. Type-A standard uncertainty calculation for V_1 and V_2

Measurement, j	V_1	$V_1 - \bar{V}_1$	$(V_1 - \bar{V}_1)^2$	V_2	$V_2 - \bar{V}_2$	$(V_2 - \bar{V}_2)^2$
1	2502.12	-76.04	5781.70	3024.27	-33.52	1123.48
2	2575.90	-2.26	5.10	3018.49	-39.30	1544.36
3	2626.51	48.35	2337.96	3040.96	-16.83	283.19
4	2638.82	60.66	3679.94	3051.80	-5.99	35.86
5	2459.82	-118.34	14003.76	2996.62	-61.17	3741.57
6	2605.77	27.61	762.45	3036.33	-21.46	460.46
7	2533.75	-44.41	1972.03	3048.63	-9.16	83.88
8	2649.21	71.05	5048.46	3117.49	59.70	3564.29
9	2574.75	-3.41	11.61	3079.14	21.35	455.89
10	2540.48	-37.68	1419.59	3128.86	71.07	5051.18
11	2630.15	51.99	2703.22	3068.75	10.96	120.16
12	2600.61	22.45	504.11	3082.12	24.33	592.03
	$\bar{V}_1$ = 2578.16		sum = 38229.94	$\bar{V}_2$ = 3057.79		sum = 17056.34
Standard deviation $s(V_1) = \sqrt{\text{sum}/11}$ = 58.95						$s(V_2)$ = 39.38
Type-A standard uncertainty $U_A(V_1)$ = 17.02						$U_A(V_2)$ = 11.37

IV. CONCLUSION

In the present work, the theoretical analyses and experimental results show that balancing of rigid rotor with the IC method could satisfy the performance and required precision. The proposed procedure has been illustrated by a series of tests with a two-plane rotor. The system of balancing discussed in this experiment was developed for balancing rotors easily and accurately, and found to be robust against measurement noise. It is essential to extract the fundamental frequency signal from the mixed-signal collected by sensors during balancing test measurement. The unbalance reduction rate could be above 95% proving that the procedure was found to be effective and practical. In particular, the application of the proposed IC method on the balancing machines can help to determine the unbalances quickly, accurately, and reliably. It can be considered as a standard procedure in calibrating the new dynamic balancing machine in production including marine applications and fisheries. Besides, some sources of uncertainty (i.e., of Type-B) should be also considered in a vibration measurement in further research.

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